Adaptive Homing is in P

Natalia Kushik and Nina Yevtushenko
Tomsk State University, Russia

Motivation

- Rigorous testing and verification of technical systems rely on using formal models

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That is why we all are here at the MBT ☺

- Reactive systems are often described with the use of Finite State Machines (FSMs)

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That is why FSMs are widely used as formal specifications for testing purposes

Motivation (2)

What if…

- The behavior of a system under test is described by a Finite State Machine
- A system under test has no reliable RESET

OR

- A RESET button is too expensive

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- Homing and synchronizing sequences can be of a big help to take a system to a known state
- After that, the known state can be considered as the initial state when applying test sequences

Motivation (3)

- Complex technical systems are mostly described with the use of nondeterministic specifications
- Nondeterminism occurs due to limited controllability and limited observability (for example)
- The complexity of problems of checking the existence and / or deriving homing and synchronizing sequences for nondeterministic machines is very high

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Complexity of deriving homing sequences for nondeterministic machines asks for decreasing

Outline

1) Finite State Machines
2) Homing Experiments with FSMs
3) Preset and Adaptive experiments
4) Adaptive Homing Problem
5) Complexity of Adaptive Homing Problem
6) Conclusions and Future Work

How to decrease the complexity when solving analysis problems for FSMs?

Utilizing scalable representations allows to ‘hide’ the complexity

Providing effective heuristics for specific types of analysis problems

Switching **from preset to adaptive** strategy

Defining specific FSM classes, when the worst complexity cannot be reached

We study (From Preset to Adaptive) & (Specific FSM classes)

What is the complexity?

Time **Space**

We focus on the problem of existence and synthesis of homing sequences for complete nondeterministic FSMs

To decrease…

1) We switch from preset to adaptive experiments

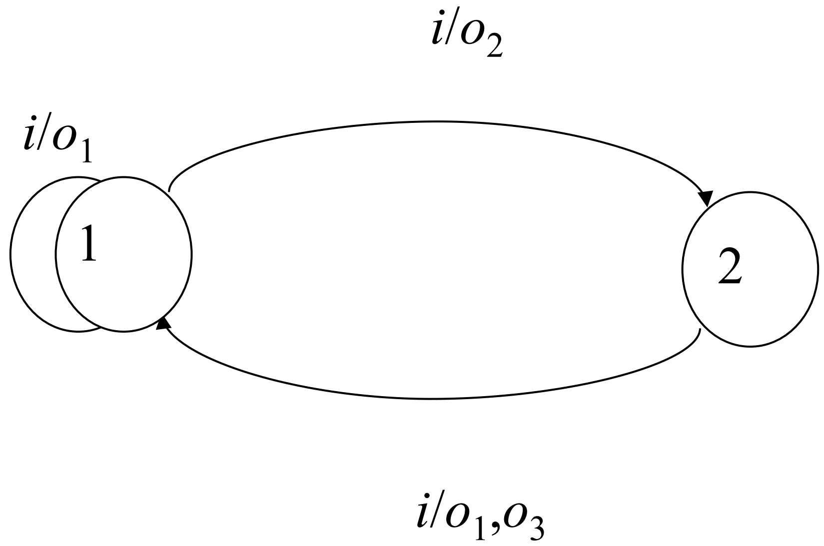
2) We define / consider specific FSM classes

Our purpose: to get the polynomial complexity in both senses, namely, for time and space

Finite State Machine (FSM)

$\mathcal{S} = (S, I, O, h_S, S_{in})$ is NFSM

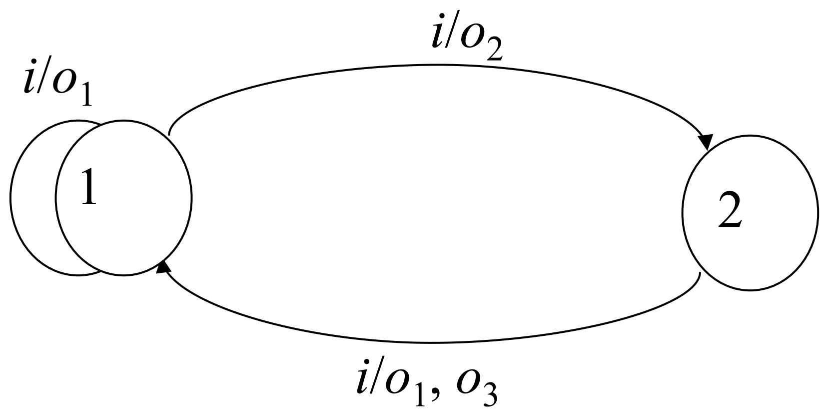
- S is a finite nonempty set of states with a nonempty subset S_{in} of initial states
- I and O are finite input and output alphabets
- $h_S \subseteq S \times I \times O \times S$ is a behavior relation



FSM $\mathcal{S} = (S, I, O, h_S, S_{in})$ can be

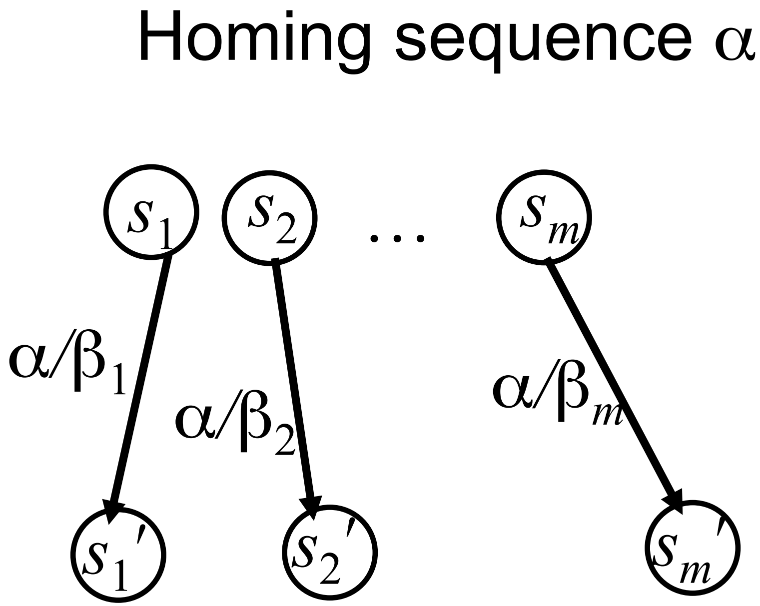
- *deterministic* if for each pair $(s, i) \in S \times I$ there exists at most one pair $(o, s') \in O \times S$ such that $(s, i, o, s') \in h_S$
  otherwise, $\mathcal{S}$ is *nondeterministic*
- *complete* if for each pair $(s, i) \in S \times I$ there exists $(o, s') \in O \times S$ such that $(s, i, o, s') \in h_S$
  otherwise, $\mathcal{S}$ is *partial*
- *observable* if for each triple $(s, i, o) \in S \times I \times O$ there exists at most one state $s' \in S$ such that $(s, i, o, s') \in h_S$
  otherwise, $\mathcal{S}$ is *nonobservable*

This one is nondeterministic, complete and observable



Homing sequence

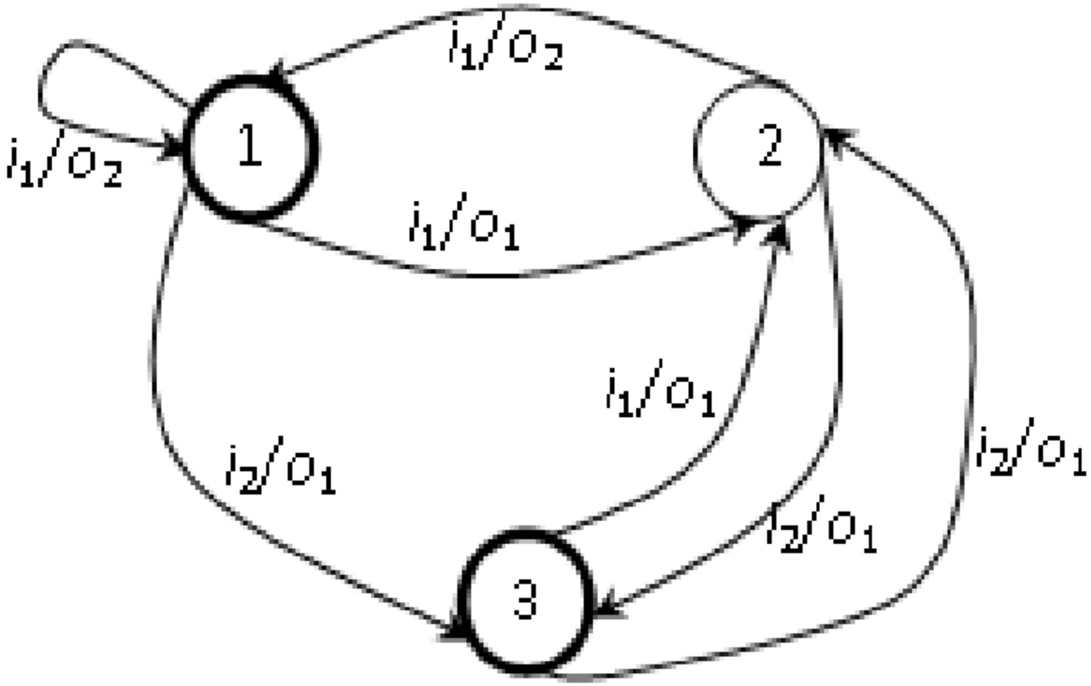
- The sequence α allows to detect the final state of the machine under experiment after α application
- After applying α at any state s_i and observing an output response β_i the final state s_i' becomes known



Homing experiment = applying α + observing β_i + drawing a conclusion about s_i'

Example

$\mathcal{S}$ = ({1, 2, 3}, {i_1, i_2}, {o_1, o_2}, h_S, {1, 3})



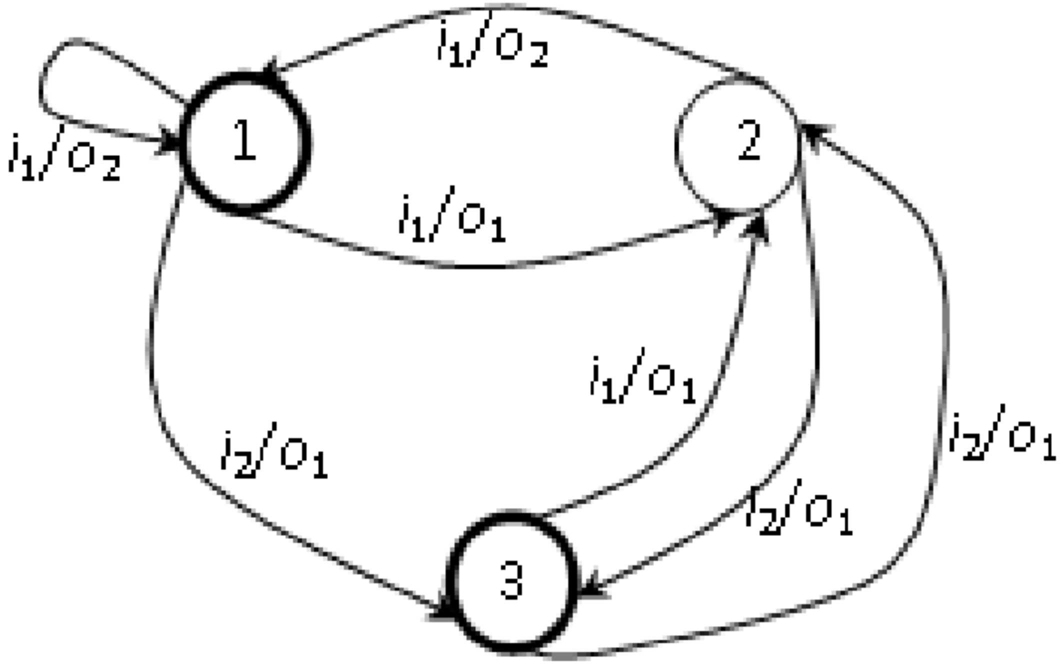
The set of initial states {1, 3} is homed by a sequence of length 1, namely, i_1

Homing Experiment: apply i_1, **if** o_1 is observed then s' = 2

else s' = 1

Example (cont-d)

$\mathcal{S} = (\{1, 2, 3\}, \{i_1, i_2\}, \{o_1, o_2\}, h_S, \{1, 3\})$



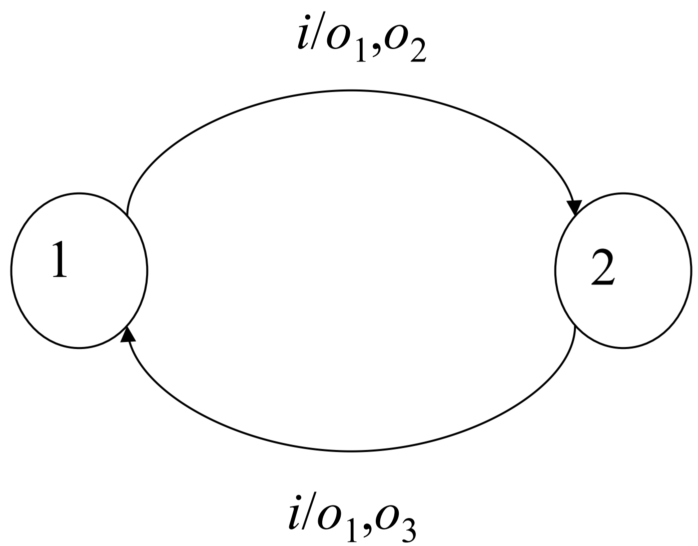
The sequence i_1 homes the set {1, 3} of states
BUT
This sequence does not home the set {1, 2, 3} of states

When a homing sequence exists?

For deterministic machines – all cool ☺

- Each complete deterministic connected reduced FSM has a homing sequence
- The length of the homing sequence for a deterministic FSM with *n* states does not exceed

  $n(n - 1)/2$
- For a nondeterministic FSM a homing sequence does not always exist



An FSM without a homing sequence

Problem of existence of a homing sequence for a complete nondeterministic FSM

Is stated as the following decision problem

HOMING
Input: complete observable nondeterministic FSM
$\mathcal{S} = (S, I, O, h, S_{in})$, $|S| = n$, $|S_{in}| = m$, $2 \le m \le n$
Question: does there exist a homing sequence for the FSM $\mathcal{S}$?

This problem is PSPACE-complete*

* N.G. Kushik, V.V. Kulyamin, N.V. Evtushenko. On the complexity of existence of homing sequences for nondeterministic finite state machines. Programming and Computer Software, 2014, Vol. 40, No. 6, pp. 331–334

The length of a homing sequence

- There exists a class of FSMs with $n > 3$ states, $(n - 1)$ inputs, and $O(n^2)$ outputs
- The length of a shortest homing sequence for any FSM of this class equals $2^{n-1} - 1$

$$\Downarrow$$

Very huge (exponential !!!) complexity

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Does not fit into the polynomial space or can be derived in the polynomial time

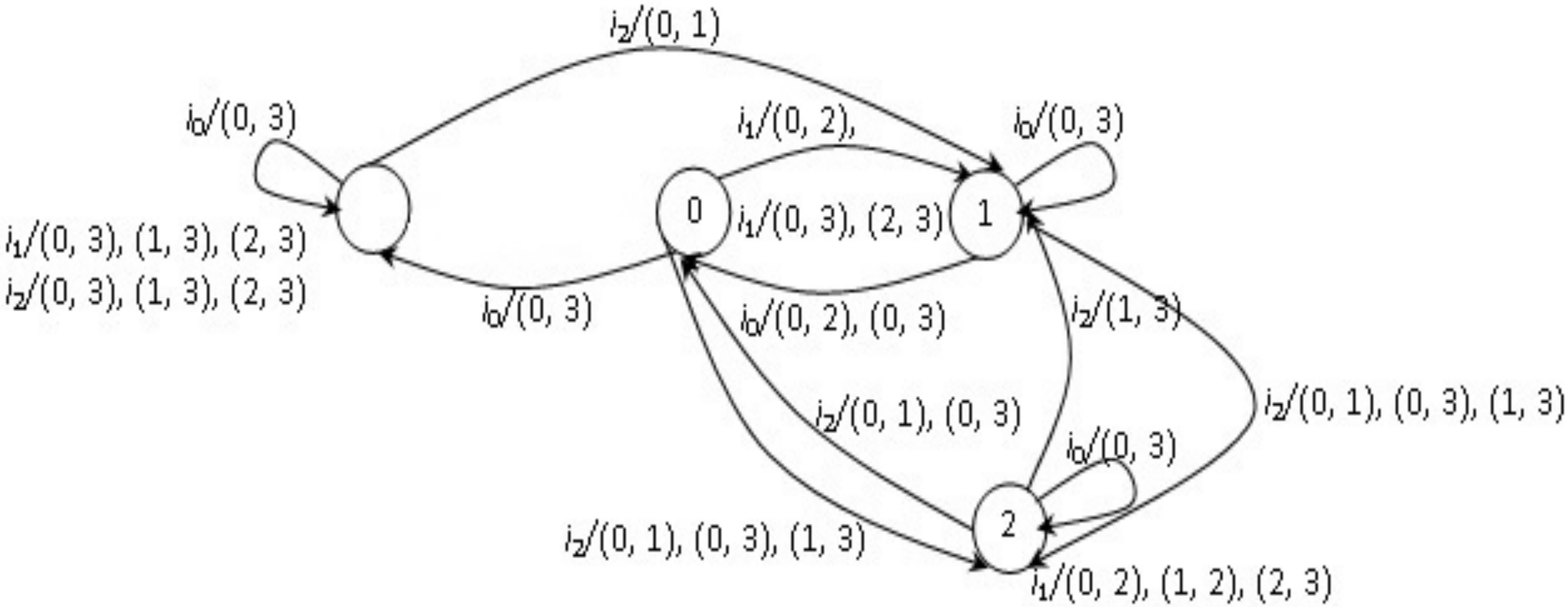
Reachability of the exponential length of a homing sequence*

- Consider a nondeterministic FSM S_n, $n > 3$, with a set $S = \{0, 1, 2, \ldots, n-1\}$ of states, a set $I = \{i_0, i_1, \ldots, i_{n-2}\}$ of inputs, and a set $O = \{(i, j): i, j = 0, \ldots, n-1 \;\&\; i < j\}$ of outputs
- The shortest homing sequence traverses a path of the truncated successor tree labeled with state pairs of a chain

  $\{0, 1, \ldots, n-1\}, \{1, 2, \ldots, n-1\}, \{0, 2, \ldots, n-1\}, \{2, 3, \ldots, n-1\}, \ldots, \{0, n-1\}$
- The length of this sequence equals $2^{n-1} - 1$

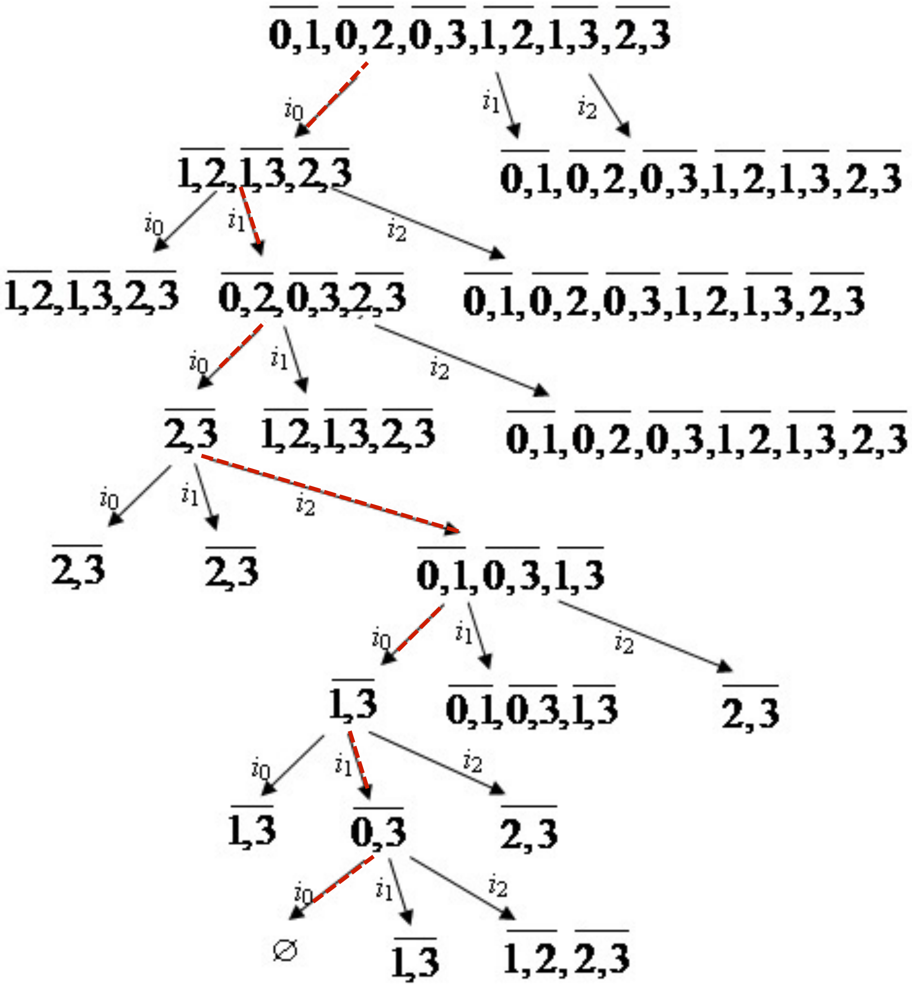
* N. Kushik, N. Evtushenko. On the Length of Homing Sequences for Nondeterministic Finite State Machines. Proc. of the CIAA, 2013, pp. 220-231

FSM S_n, $n = 4$



i2/(0, 1)
i0/(0, 3)
i1/(0, 2),
i0/(0, 3)
0
1
i1/(0, 3), (1, 3), (2, 3)
i2/(0, 3), (1, 3), (2, 3)
i1/(0, 3), (2, 3)
i0/(0, 3)
i0/(0, 2), (0, 3)
i2/(1, 3)
i2/(0, 1), (0, 3), (1, 3)
i2/(0, 1), (0, 3)
i0/(0, 3)
2
i2/(0, 1), (0, 3), (1, 3)
i1/(0, 2), (1, 2), (2, 3)

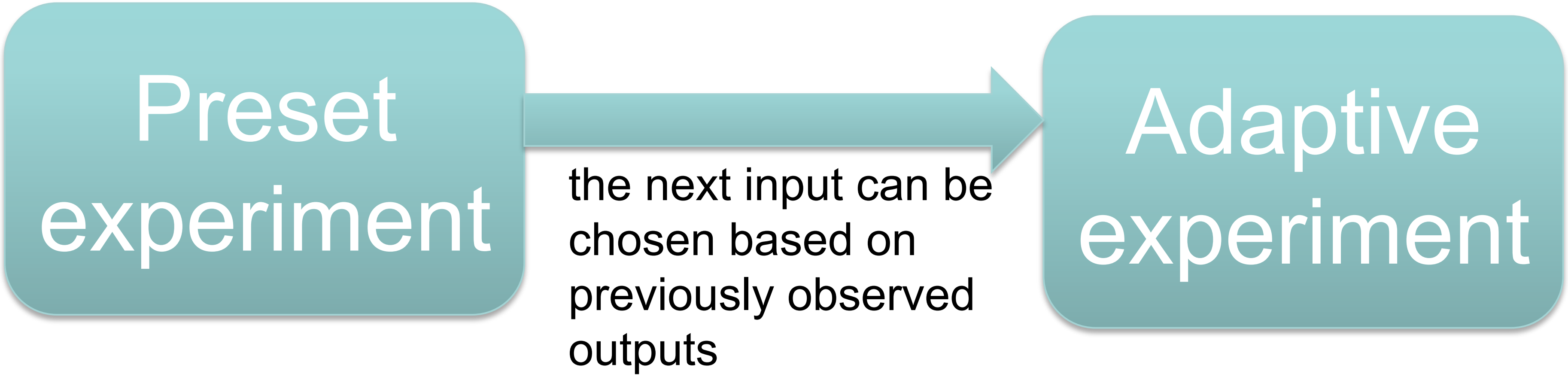
S_4 truncated successor tree



A shortest homing sequence traverses a chain of state subsets

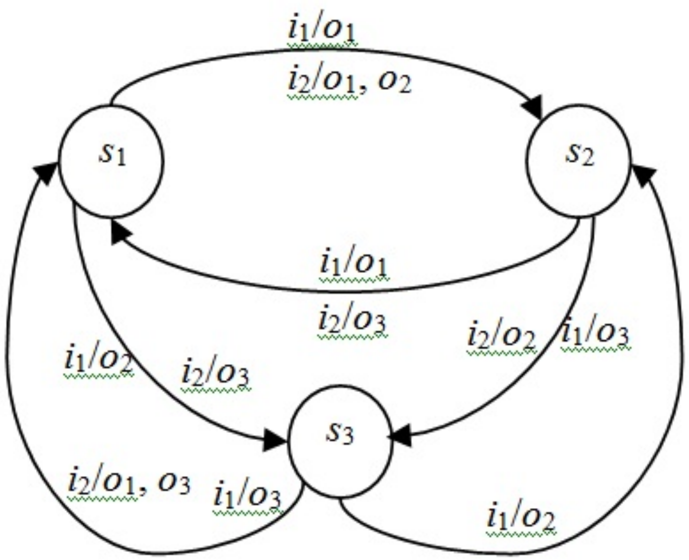
{0, 1, 2, 3}, {1, 2, 3}, {0, 2, 3}, {2, 3}, {0, 1, 3}, {1, 3}, {0, 3}

How to decrease the complexity through adaptive experiments

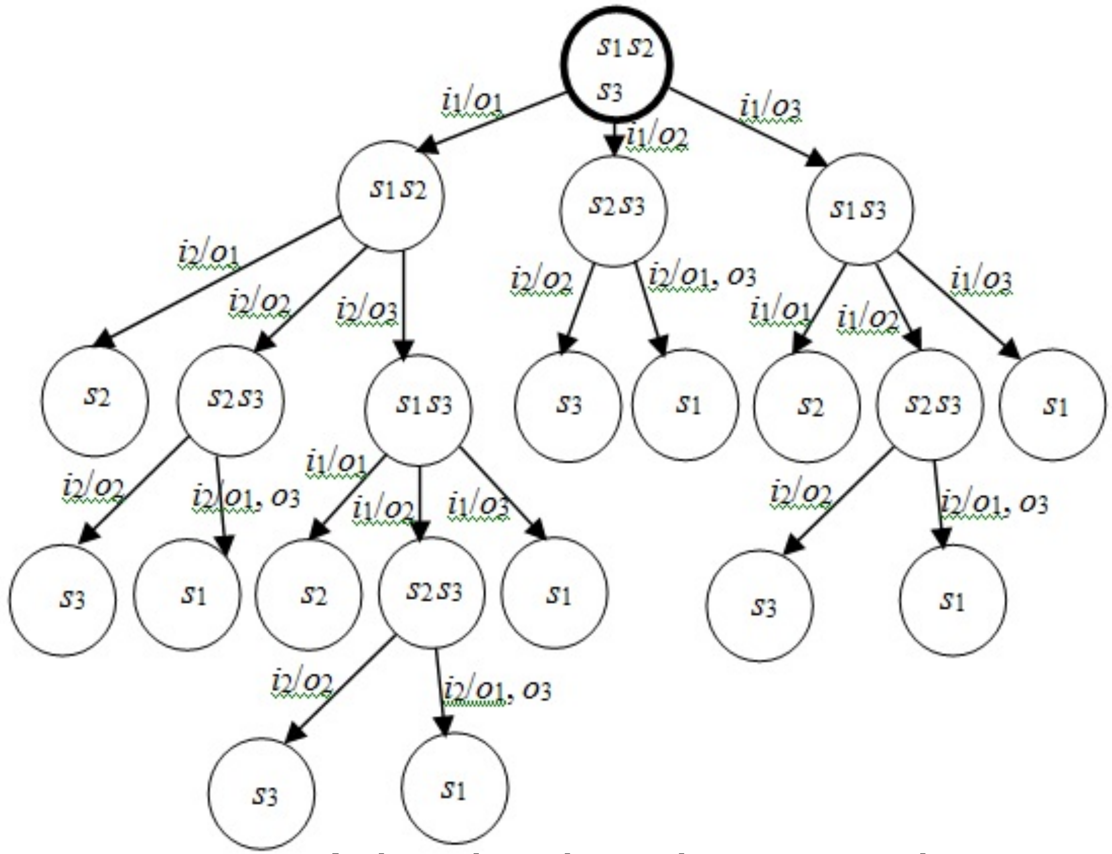


- The experiment is now represented by a ***Test Case***
- A ***Test Case*** is a connected single-input output-complete observable initialized FSM with the acyclic transition graph

Example for homing experiment



Nondeterministic FSM



Adaptive homing experiment
for nondeterministic FSM

Problem of existence of a homing test case for a complete nondeterministic FSM

ADAPTIVE HOMING problem

But only for the class of NFSMs where $S = S_{in}$

Input: complete observable nondeterministic FSM
$\mathcal{S} = (S, I, O, h)$, $|S| = n$
Question: does there exist a homing test case for the FSM $\mathcal{S}$?

The problem can be reduced to that of checking if there exists a homing test case for each pair of FSM states

Theorem. FSM $\mathcal{S}$ has a homing test case, if and only if each pair of FSM states is adaptively homing

Problem of existence of a homing test case for a complete nondeterministic FSM (2)

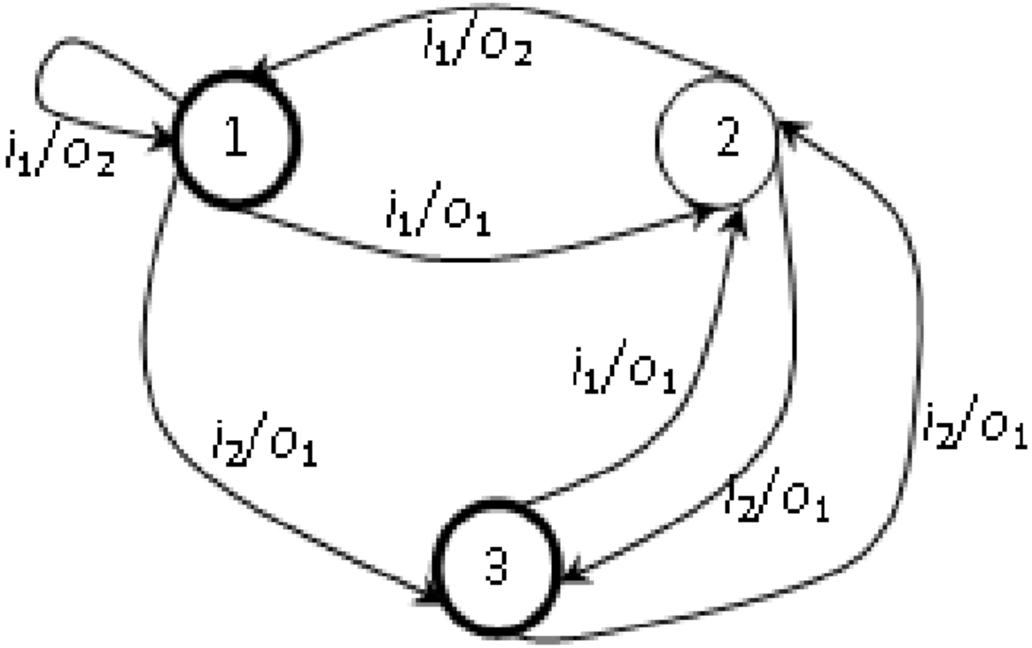
$\mathcal{S} = (S, I, O, h)$, $|S| = n$

Proposition. A pair of states s_1 and s_2 of an FSM $\mathcal{S}$ is adaptively homing, if and only if the intersection $\mathcal{S}/s_1 \cap \mathcal{S}/s_2$ does not have a complete submachine

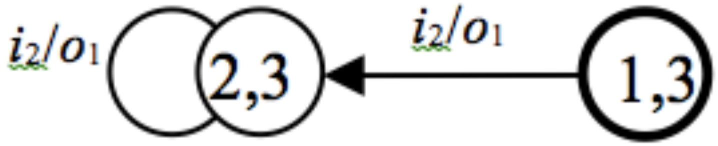
Proposition. Adaptive Homing Problem for a pair s_1 and s_2 of FSM states is in P complexity class, if the number of FSM inputs/outputs is polynomial w.r.t. the number n of FSM states

Theorem. Adaptive Homing Problem for a complete observable FSM $\mathcal{S} = (S, I, O, h)$, $|S| = n$, $S = S_{in}$, is in P complexity class

Intersection



An FSM $\mathcal{S}$



An intersection $\mathcal{S}/1 \cap \mathcal{S}/3$

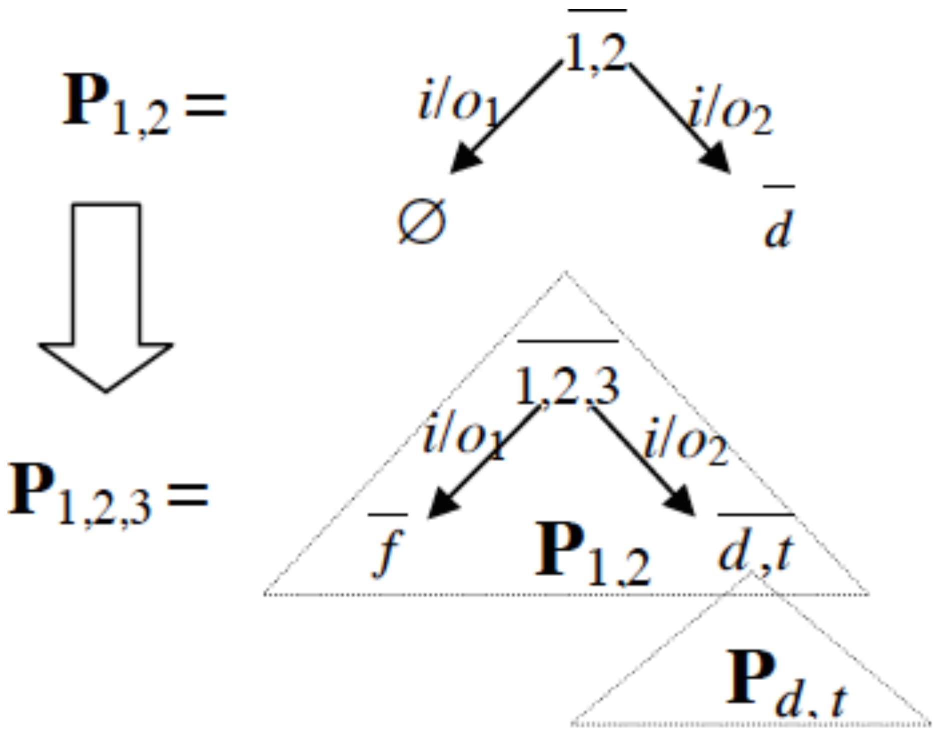
The intersection $\mathcal{S}/1 \cap \mathcal{S}/3$ does not have a complete submachine

The intersection $\mathcal{S}/s_1 \cap \mathcal{S}/s_2$ captures all the traces that are defined in the FSM $\mathcal{S}$ at states s_1 and s_2

Deriving homing test cases for nondeterministic FSMs

Given an FSM $\mathcal{S} = (S, I, O, h)$, $|S| = n$, such that each state pair (i, j) is adaptively homing

- We build the test case iteratively starting from the pair (1, 2) of states
- We add other states one by one, to the set of initial states (the root of the tree)
- Test cases of a type $\mathbf{P}_{i,j}$ are used at each step



Proposition. The height of the homing test case for $\mathcal{S}$ does not exceed $O(n^3)$

The class of nice machines is not empty

- If the preset homing experiment exists for an FSM, then this FSM has an adaptive homing test case
- Each complete deterministic connected reduced FSM has a homing sequence
- A preset homing sequence homes each pair of FSM states

$$\Downarrow$$

- Each complete deterministic connected reduced FSM is adaptively homing

Conclusions

- We prove that for each $n > 2$ there exists a class of complete nondeterministic FSMs, such that Adaptive Homing problem for an FSM of this class is in P complexity class
- The maximal height of a homing test case for an FSM from this class is of the order $O(n^3)$

- For now

Necessary and sufficient conditions of existence of a homing test case for an FSM are established only for the case, when $S = S_{in}$

Future work

- We plan to estimate, how practical is the class of nondeterministic FSMs that we have just discussed

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Which technical systems can be described by nondeterministic FSM such that each pair of FSM states is adaptively homing?

- We plan to check what happens when $|S_{in}| < |S|$

Thank you!